

Statistics Formula Sheet

Every key formula on one sheet, each with a free calculator at dackohn.com/stats · Critical values computed with SciPy

Descriptive Statistics

Mean
$\bar{x} = \sum x_i / n$
Median
middle value of sorted data (average of the two middle values if n is even)
Sample variance
$s^2 = \sum (x_i - \bar{x})^2 / (n - 1)$
Sample std deviation
$s = \sqrt{s^2}$
Population variance
$\sigma^2 = \sum (x_i - \mu)^2 / N$
Coefficient of variation
$CV = s / \bar{x} \times 100\%$
Interquartile range
$IQR = Q_3 - Q_1$
Outlier fences
below $Q_1 - 1.5 \cdot IQR$ or above $Q_3 + 1.5 \cdot IQR$

Probability

Complement
$P(A') = 1 - P(A)$
Union (OR)
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Intersection, independent
$P(A \cap B) = P(A) \cdot P(B)$
Conditional
$P(A B) = P(A \cap B) / P(B)$
Bayes' theorem
$P(A B) = P(B A) \cdot P(A) / P(B)$
Law of total probability
$P(A) = \sum P(A B_i) \cdot P(B_i)$
Expected value / variance
$E[X] = \sum x \cdot P(x), \text{Var}(X) = E[X^2] - (E[X])^2$

Counting

Factorial
$n! = n \cdot (n-1) \cdots 1, 0! = 1$
Permutations (order matters)
$nPr = n! / (n - r)!$
Combinations (order doesn't)
$nCr = n! / (r! \cdot (n - r)!)$

Distributions

Z-score (standardize)
$z = (x - \mu) / \sigma$
Empirical rule (normal)
$\pm 1\sigma: 68.3\% \cdot \pm 2\sigma: 95.4\% \cdot \pm 3\sigma: 99.7\%$
Binomial
$P(X=k) = C(n,k) p^k (1-p)^{n-k}$
Binomial mean / var
$\mu = np, \sigma^2 = np(1-p)$
Poisson
$P(X=k) = e^{(-\lambda)} \cdot \lambda^k / k!, \mu = \sigma^2 = \lambda$

Standard Errors

Mean
$SE = s / \sqrt{n}$
Proportion
$SE = \sqrt{(\hat{p}(1-\hat{p}) / n)}$
Difference of two means (Welch)
$SE = \sqrt{(s_1^2/n_1 + s_2^2/n_2)}$
Difference of two proportions
$SE = \sqrt{(\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2)}$
SD describes the data; SE describes the precision of an estimate.

Confidence Intervals

Mean (σ unknown)
$\bar{x} \pm t^* \cdot s/\sqrt{n}, df = n - 1$
Mean (σ known)
$\bar{x} \pm z^* \cdot \sigma/\sqrt{n}$
Proportion (Wald)
$\hat{p} \pm z^* \cdot \sqrt{(\hat{p}(1-\hat{p})/n)}$
Proportion (Wilson, better near 0 or 1)
$[\hat{p} + z^2/2n \pm z\sqrt{(\hat{p}(1-\hat{p})/n + z^2/4n^2)}] / (1 + z^2/n)$
Common z^*
90% $\rightarrow 1.645 \cdot 95\% \rightarrow 1.960 \cdot 99\% \rightarrow 2.576$

Hypothesis Tests

Z-test (mean)
$z = (\bar{x} - \mu_0) / (\sigma/\sqrt{n})$
One-sample t
$t = (\bar{x} - \mu_0) / (s/\sqrt{n}), df = n - 1$
Two-sample t (Welch)
$t = (\bar{x}_1 - \bar{x}_2) / \sqrt{(s_1^2/n_1 + s_2^2/n_2)}$
Paired t
$t = \text{mean}(d) / (s_d/\sqrt{n}), df = n - 1$
Two-proportion z
$z = (\hat{p}_1 - \hat{p}_2) / \sqrt{(\hat{p}(1-\hat{p}))(1/n_1 + 1/n_2)}$
Decision rule
reject H_0 if $p \leq \alpha$

Sample Size

Estimate a mean to $\pm E$
$n = (z^* \cdot \sigma / E)^2$
Estimate a proportion to $\pm E$
$n = z^{*2} \cdot \hat{p}(1-\hat{p}) / E^2$ ($\hat{p} = 0.5$ if unknown)
Compare two means (per group)
$n = 2(z_{1-\alpha/2} + z_{1-\beta})^2 \sigma^2 / \Delta^2$
Power constants
80% power $\rightarrow z_{1-\beta} = 0.842 \cdot 90\% \rightarrow 1.282$
Classic result: $\pm 5\%$ at 95% confidence needs $n = 385$ responses.

Nonparametric

Mann-Whitney U
$U_1 = R_1 - n_1(n_1+1)/2, U = \min(U_1, U_2), U_1 + U_2 = n_1 n_2$
Spearman's ρ (no ties)
$\rho = 1 - 6\sum d^2 / (n(n^2 - 1))$
Use when data is ordinal or clearly non-normal with a small sample.

Correlation & Regression

Pearson r

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{[\sum(x_i - \bar{x})^2 \cdot \sum(y_i - \bar{y})^2]}}$$

Test of r

$$t = r\sqrt{(n-2) / \sqrt{(1-r^2)}}, df = n-2$$

Regression line

$$\hat{y} = b_0 + b_1x$$

Slope / intercept

$$b_1 = r \cdot (s_y / s_x), b_0 = \bar{y} - b_1\bar{x}$$

Fit

$$R^2 = 1 - SS_{res} / SS_{tot}$$

Effect Size

Cohen's d

$$d = (\bar{x}_1 - \bar{x}_2) / s_{pooled}$$

Benchmarks (d)

0.2 small · 0.5 medium · 0.8 large

Benchmarks (r)

0.1 small · 0.3 medium · 0.5 large

ANOVA

$$\eta^2 = SS_{between} / SS_{total}$$

Odds ratio (2x2: a b / c d)

$$OR = (a \cdot d) / (b \cdot c)$$

Relative risk

$$RR = [a/(a+b)] / [c/(c+d)]$$

χ^2 & ANOVA

Chi-square

$$\chi^2 = \sum (O - E)^2 / E$$

Expected count (independence)

$$E = (\text{row total} \times \text{column total}) / n$$

Degrees of freedom

$$df = (\text{rows} - 1)(\text{columns} - 1)$$

Cramér's V

$$V = \sqrt{(\chi^2 / (n \cdot (k-1)))}, k = \min(\text{rows}, \text{cols})$$

ANOVA F-ratio

$$F = MS_{between} / MS_{within}$$

Agreement & Reliability

Cohen's κ (2 raters)

$$\kappa = (p_o - p_e) / (1 - p_e)$$

Fleiss' κ (3+ raters)

$$\kappa = (P_o - P_e) / (1 - P_e), P_o = \text{mean pairwise agreement}$$

Krippendorff's α

$$\alpha = 1 - D_{observed} / D_{expected}$$

Rough guide (Landis & Koch)

0.41-0.60 moderate · 0.61-0.80 substantial · >0.80 almost perfect

Multiple Comparisons

Chance of ≥ 1 false positive in m tests

$$1 - (1 - \alpha)^m \text{ (20 tests at 0.05 } \rightarrow 64\%)$$

Bonferroni

reject if $p \leq \alpha / m$

Holm (step-down)

sort p; reject $p_{(i)} \leq \alpha / (m - i + 1)$ until the first failure

Benjamini-Hochberg (FDR)

largest i with $p_{(i)} \leq (i/m) \cdot \alpha$; reject 1...i

ML & AI Evaluation

Precision / recall

$$P = TP / (TP + FP), R = TP / (TP + FN)$$

F1 score

$$F1 = 2PR / (P + R) = 2TP / (2TP + FP + FN)$$

McNemar (two models, same items)

$$\chi^2 = (|b - c| - 1)^2 / (b + c)$$

pass@k (n samples, c correct)

$$\text{pass@k} = 1 - C(n-c, k) / C(n, k)$$

Expected calibration error

$$ECE = \sum (n_b / N) \cdot |acc_b - conf_b|$$

Elo win probability

$$P(A \text{ beats } B) = 1 / (1 + 10^{((R_B - R_A)/400)})$$

Common Critical Values

z*, two-sided

90%: 1.645 · 95%: 1.960 · 99%: 2.576

z*, one-sided 95%

1.645

t*, 95% two-sided

df 5: 2.571 · 10: 2.228 · 20: 2.086 · 30: 2.042

χ^2 , $\alpha = 0.05$

df 1: 3.841 · 2: 5.991 · 3: 7.815 · 4: 9.488

Computed with SciPy. Full tables below.